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# Electric breakdown of dielectric membranes via axisymmetric necking

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## Abstract

It is well-known that when a dielectric membrane coated with electrodes is subject to an electric field through its thickness and a dead-load all-round tension at its edge, the nominal electric field versus the nominal displacement typically has a maximum (or a limit point) (1). Based on this fact, it is commonly believed that when this maximum is reached during electric loading, rapid but uniform thickness-thinning will take place that eventually leads to electric breakdown. However, recent research on the tube inflation problem has shown that although a limit point also exists in the internal pressure versus the volume, the tube does not always snap through to another uniformly inflated configuration with a larger radius; other possibilities also exist depending on how inflation is conducted (2). Equipped with this knowledge, we have recently carried out an in-depth study on the bifurcation behaviour of the above-mentioned dielectric membrane (3). In particular, we have explored the possibility of electric breakdown via axisymmetric necking (4, 5) and have shown that the condition for axisymmetric necking is different from the condition for the limit-point instability. It is further shown that for a class of free-energy functions, axisymmetric necking may compete with the limit-point instability and the Treolar-Kearsley instability (whereby equal nominal forces result in unequal stretches in the membrane plane), and become the preferred mode of instability depending on how the membrane is loaded.

In this talk, we present our latest results on the development of a one-dimensional (1d) reduced model for characterising axisymmetric deformations of a dielectric membrane (including axisymmetric necking). Our reduction is built on the variational asymptotic method (6,7), so that the resulting 1d model is asymptotically self-consistent. The 1d model offers an easier and more efficient way to analyze axisymmetric necking in a dielectric membrane in the linear, weakly nonlinear and fully nonlinear regimes. It delivers results identical to the 3d theory in the linear and weakly nonlinear regimes, and near-identical results in the fully nonlinear regime. We demonstrate the straightforward implementation of the 1d model by solving it using the Rayleigh–Ritz method and validate it by comparison with finite-element simulations. The 1d model enables a precise calculation of the minimum thickness that a dielectric membrane can reach when

necking instability occurs and the effects of imperfections so that the integrity of a dielectric elastomer actuator can be assessed with respect to electric breakdowns. The developed methodology is not problem-specific and so is also applicable to other soft materials subjected to any other fields.

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