
Relating local field fluctuations in composites with the sensitivity of their effective response to constitutive parameters: application to identification of elastic and viscoelastic materials.

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Abstract

In the study of the behaviour of composite materials, micromechanical models and homogenization methods make it possible to predict their effective macroscopic responses from the knowledge of their geometric and material properties at the micro-scale. The fundamental components of micromechanical homogenization models are the microstructure, phase properties, and the homogenization scheme itself. While advancements in tomographic imaging and image processing have made microstructural data more easily accessible, determining the constitutive properties of the constituents remains challenging in experimental conditions. An alternative to in situ microscale experimentations is the inverse homogenization process, wherein microscale parameters are inferred from macroscale experimental data. Also referred to as reverse engineering or de-homogenization, this method revolves around minimizing, relative to the microscale properties, the discrepancy between measured macroscale data and simulated data so as to identify the former. Two types of macroscale data can be used: (i) In cases where the macroscale behaviour is homogeneous, average variables, such as effective elastic moduli, are employed (9). Alternatively, (ii) structural experiments, which inherently produce heterogeneous macroscale data, can be used (7). In these cases, displacement fields are typically measured via digital image correlation, and multiscale computational methods, such as FE² techniques (2), are applied to account for the heterogeneities at both the macro and microscales. While this latter approach is efficient, as it generates a larger volume of data from a single test, it is often more complex to set up.

As highlighted by (4), inverse homogenization constitutes an ill-posed problem whose solution existence, uniqueness, and stability are not guaranteed. In this context, significant efforts have been made to analyse structural experiences in order to extract more information and therefore enhance the likelihood of successful parameter identification. Noticeably, some gradient-based algorithms for cost functionals minimization, using gradient approximations by finite differences, as well as semi-analytical methods have been developed (3). Given the cost of accurate gradient computations, gradient-free algorithms have also been explored (5),

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along with techniques related to model-reduction (6). Using a Monte Carlo approach, (8) examined the impact of noise in displacement fields on the identified properties, depending on the specific structural experiment. Both linear and nonlinear identified behaviours have been investigated in the literature.

In the present work, we focus on the identification of a subset of constitutive parameters in a context where the macroscale heterogeneity is neglected. The unknown microscale parameters are to be identified from the effective response of the composite and the known microscale parameters. The solution is obtained by minimizing a cost function which compares the measured effective response to the one obtained numerically with guessed parameters and using full-fields homogenization computations. In order to use gradient-descent algorithms, the gradient and the Hessian matrix of the cost function need to be computed, through the derivatives of the effective response itself relatively to the microscale parameters. To this end, a second-order asymptotic expansion of the effective energy is proposed, building on the developments in (1). The first-order derivatives are shown to be directly computable from the strain fields generated by the homogenization simulations. However, a few additional simulations are required for the Hessian matrix. This approach is also particularized when the effective potential can be written as a function of effective parameters as for linear elastic or viscoelastic phases using the Laplace-Carson transform. In the general non-linear case, the method is described in terms of a stress- or energy-based formulation. We provide first an analytical example to highlight some key issues related to the inverse method: the solution existence, uniqueness and stability. A second example consists in identifying the transversely isotropic elastic behaviour of carbon fibres embedded within an isotropic elastic matrix based on full-field simulations. The gradient and the Hessian matrix are computed, and a linearization of the uncertainties is performed to quantify the propagation of perturbations on each input parameter and thus determine the relevant experiments to be planned in order to improve the identification procedure. Finally, a last example concerns the identification of the viscoelastic behaviour of a matrix in a glass-fibre composite (isotropic elastic fibre), again, with full-field computations. All the examples presented here are based on synthetic data, while circumventing the so-called inverse crime, in order to illustrate the proposed approach. The satisfying performances of the latter will be illustrated quantitatively in terms of accuracy and numerical efficiency. In addition, the relevance of the linearized uncertainty quantification analysis will be highlighted.

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