Tencers: Tension-Constrained Elastic Rods

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Abstract

In this talk I will discuss a class of lightweight structures that we coin tencers, short for tension-constrained elastic rods. Tencers consist of initially straight, flexible rods that are shaped through inextensible cables connected to the rods or external anchor points. Such a bending-active structure is in static equilibrium when the tension forces in the cables exactly cancel the bending, stretching, and twisting forces in the rods. The resulting 3D shapes of the rods then emerge as a consequence of all the forces acting on them. Our formulation supports arbitrary rod cross-sections and material properties. Rods can be open or closed, knotted, and arranged in arbitrary topologies.
We specifically focus on equilibrium states with no contacts among rods, i.e. each rod is only constrained by cables but no self-contacts or contacts with other rods. These configurations can be considered as a generalization of tensegrities, which are commonly composed of rigid rods in compression and flexible cables in tension. We show how this generalization to flexible rods leads to a rich design space, where complex target shapes can be achieved even with a small set of rods and cables.

However, allowing the rods to deform fundamentally changes the underlying kinematics and thus warrants a different computational approach than those commonly used for tensegrities. To explore the space of tencers, we present an inverse design optimization algorithm that solves for the length and placement of cables such that the equilibrium state of the rod network best approximates a given set of input curves. This algorithm continuously tracks the equilibrium state of the tencer while the geometric and physical parameters of the structure are optimized. For this purpose, we employ an efficient simulation method based on the discrete elastic rods formulation in the inner loop of the optimization.

We introduce appropriate sparsity terms to minimize the number of required cables, which significantly simplifies fabrication. Using our algorithm, we explore new classes of bending-active 3D structures, including elastic tensegrity knots that only require a few internal cables. We design and fabricate several physical models from basic materials that attain complex 3D shapes with unique structural properties.

Tencers are intriguing mechanical structures that find applications across scales, from lightweight structures in architecture to building blocks of mechanical metamaterials. Tencers commonly exhibit complex buckling and multistability. Even in the simplest examples, we observe interesting geometric and mechanical behavior. For example, an elastic ring buckles into 3D as the length of a cable spanning its diameter is reduced. As this length approaches zero, the rod flattens into a self-intersecting eight-shaped configuration.

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Beyond their practical relevance, tencers are also interesting objects of theoretical study. We mention a few open questions: For an elastic knot of a specific knot type, what is the minimal number of cables needed to separate all self-contacts? When does a tencer have multiple stable equilibrium states? Can a transition between such states be achieved without inter-penetrations of rods or cables?

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