The fracture pattern of masonry dones: a shell-ring analogy

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Abstract

Studying the fracture of masonry domes has a long track record, and findings related to the crack evolution are of immediate interest in conservation or the design of new structures. The cracking of brittle shells became famous with the investigation of the St Peter Basilica dome by G. Poleni in the eighteenth century (1). In the second half of the twentieth century, the problem was revisited by J. Heyman (2) in the context of limit plastic analysis. Due to the low tensile strength of the material and the load-bearing mechanism of the cupola, the appearance of cracks produces a hoop-stress-free equilibrium state of the structure. Heyman also suggested that further investigation of the emergence of the pattern in the cupola would be of interest (2). In particular, we seek to determine the position and order of appearance of the meridional cracks and clarify how existing cracks affect the formation of further ones, creating the whole pattern. For this purpose, we focus on the canonical structural model of a hemispherical dome whose equator is constrained to deform in its plane and consider a quasi-static increase of the self-weight of the structure. Modeling the dome as a shell under Kirchhoff-Love hypotheses, we first show that the equations governing the displacement of the dome's middle surface within the equatorial plane can be interpreted as the equations governing the deformation of a planar elastic ring, loaded by an outward pointing inner pressure and constrained by springs, a problem already investigated (3). Relationships between the ring and the dome parameters are given and found to depend strongly on the imposed boundary conditions on the shell edge and the elastic response of the dome. These results should open new ways to assess the genesis of the pattern in domes. In particular, the proposed formulation allows some freedom in modeling the top part of the dome in the spirit of Poleni's work. Finally, we turn to a numerical parametric study of the cracking pattern appearance in rings. Assuming the material to be unilateral in compression, we rely on an energetic criterion according to Griffith's postulate. We find that the tangential to radial spring stiffness ratio is a crucial driver of pattern formation by controlling elaborate bifurcations that determine the emerging crack's position. Numerical experiments are proposed and compared with finite element simulations.

References:

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(2) Heyman, J. (1988). Poleni's problem. *Proceedings of the Institution of Civil Engineers*, 84(4), 737–759.

(3) Michel, S., Sipos, A. A. (2023). Fragmentation of inflated elastic brittle rings: Emergence of the quasi-equidistant spacing of cracks. *Journal of the Mechanics and Physics of Solids*, 179, 105372.