Creasing on finite domains and its connection to Biot's surface instability

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Abstract

Over the past decades the applications of soft polymeric materials have exploded. A yet unresolved phenomenon is how and when a soft solid loses stability under compression to form a sharp inwards localisation (a crease with self-contact) at a free surface. Despite many years of research on surface instabilities in soft solids, there is still no consensus on how creases form, at what compressive strain this occurs, the nature of the instability (local or global bifurcation) and, crucially, its connection to the classical and periodic Biot instability. This talk presents results from a finite element (FE) analysis conducted in an in-house code that combines higher-order elements with numercial continuation and branch-switching algorithms to study, in detail, the nature of the creasing instability on finite domains. Assuming a quasi-incompressible Neo-Hookean material model, the findings are as follows. The fundamental path of uniform compression loses stability, i.e. the tangent operator of the force balance loses determinacy, at the classical Biot instability of 0.455 compressive strain. The larger the dimension of the finite-sized FE analysis in the loading direction (the model length), the greater the degeneracy of the bifurcation point, i.e. the longer the model, the greater the number of critical eigenmodes at the bifurcation point. The critical eigenvectors at this compound bifurcation point always take integer multiples of cosine half-waves along the model length starting at one single half wave, irrespective of the overall model length. Hence, as the model length tends to infinity, as is the case in Biot's original halfspace problem, the wavelengths of the critical eigenmodes appropriately tends to zero. Branch switching at the degenerate bifurcation point using only pure modes (one eigenmode at a time) leads to highly unstable post-critical paths (sub-critical bifurcation) where the periodic eigenmodes very quickly localise into various creases located at the minima of the periodic eigenmodes. Thus, after only a short distance along any post-critical path the eigenmode with one half-wave localises into half of a crease at either loaded end of the model; the eigenmode with two half-waves localises into a central crease or two half-creases at either end of the model; and so on.

Interestingly, all post-critical paths are sharply subcritical and lead to material self-contact at approximately 0.36 compressive strain-the experimental value often reported for the creasing instability. However, the post-critical equilibrium paths that fall from the Biot instability (0.455 strain) and lead to a self-contacting crease (0.36 strain) do not converge with increasing mesh refinement and neither does the depth of the crease localisation. Indeed, with increasing mesh refinement the crease depth tends to zero, even though the compressive strain at which self-contact occurs remains the same at 0.36. Related observations on mesh refinement by others in the literature has led to the conclusion that the self-contacting crease

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must therefore be a new type of instability that branches from the fundamental path of pure compression at a strain of 0.36, but which is not picked up by the degeneracy of the tangent operator. An alternative hypothesis considered here, which maintains a more classical flavour, is that the vanishing depth of the localisation with mesh refinement is indicative of a singularity that needs to be renormalised by additional physics. By adding surface energy (surface tension) into the model the post-critical paths do converge with mesh refinement and the greater the ratio of surface energy to shear modulus, the less sharp the sub-critical paths that emanate from the Biot instability. Finally, stochastic analysis are conducted by seeding surface imperfections into the model in the form of Gaussian random fields, which illustrate that imperfections alone can account for the difference between Biot's instability of 0.455 compressive strain derived from a perfect model, and the creasing strain of 0.36 observed in inherently imperfect experiments.

In conclusion, creasing in various forms is shown to occur as localisations along unstable post-critical paths that branch from the compound bifurcation of the Biot instability and that the post-critical regime needs to be renormalised by surface energy to give physically meaningful results. Then, the lower creasing strain observed in experiments compared to Biot's prediction takes on a classical flavour since it is governed by the subcriticality and imperfection sensitive response of the problem.