Symmetries of the Stiffness Tensor Induced by the Jaumann Rate of the Stress

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Abstract

In Continuum Mechanics in general and in Nonlinear Elasticity in particular, it is important to define *objective* rates of the Cauchy stress or the Kirchhoff stress, the latter being particularly important in Computational Mechanics. Objectivity means invariance under any arbitrary diffeomorphism (Marsden and Hughes, 1983), a differentiable map with a differentiable inverse.
The Jaumann rate (also known as Zaremba-Jaumann rate; Zaremba, 1903; Jaumann, 1905) is extensively used to define stress rates because it is not only objective, but also *corotational* with respect to the spin tensor (the skew-symmetric part of the velocity gradient). A time-dependent basis is called co-rotational if the time derivative of each of the basis vectors is obtained locally via Poisson's theorem, i.e., as the spin tensor applied as a linear map on the basis vector. In a spin-tensor-corotational basis, the components of the Jaumann rate of the stress tensor are the same as the components of the regular substantial (material) derivative of the stress tensor (Neff et al., 2024ab; Palizi et al., 2020). This makes the Jaumann rate particularly convenient from the computational point of view.

This work (Federico et al., 2024) has three main objectives.

First, we review the properties of the Jaumann rate in a completely covariant setting, and show its relation with the Lie derivative (Marsden and Hughes, 1983; Federico, 2022).

Second, we review the symmetries of the stiffness tensors induced by the Jaumann rate. These tensors generally possess both minor symmetries (i.e., invariance under exchange of the indices in the first pair and the indices in the second pair, separately) but, while major symmetry (invariance under exchange of the first pair of indices with the second) is achieved for the stiffness tensor related to the Kirchhoff stress, it is not for the stiffness tensor relative to the Cauchy stress. Some of these symmetries were missed in some past work (Palizi et al., 2020).

Third, we provide an absolute expression of both stiffness tensors, in terms of an expression not involving contractions with arbitrary second-order tensors. These absolute expressions, both in component-free form and component form, are particularly useful for numerical applications.

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