Non-Hamiltonian symmetry for coupled Ziegler pendulums

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Abstract

The Ziegler pendulum is a classical dynamical system introduced by Ziegler (1), that has been subject to deep studies by mathematicians, physicists and engineers in the last decades. The system is well-known for the paradoxical destabilization of the system when subject to damping (2-6), while in general a non-conservative system gains stability in presence of a dissipative force; furthermore, such as the classical double pendulum (7-9), the system shows itself to be chaotic. Recently, it has been proved in (10) that also a simpler version of the system exhibits chaotic motion for a general choice of initial conditions and parameters, while regular periodic solutions are found in presence of Hamiltonian and non-Hamiltonian symmetries. The authors presented in (11) analytical and numerical results concerning several variants of the Ziegler pendulum, including the presence of gravity, linear elastic potentials and friction in two possible formulations, in order to analyze the surviving or breaking of the integrable cases found in (10). Furthermore, a study of a discrete version has been performed through explicit calculations on the sets of periodic points, suggesting that the discrete map associated to the Ziegler pendulum does not satisfy the definition of chaotic map in the sense of Devaney (12) for a choice of the parameters that corresponds to a general case of chaotic motion for the original system. This last result is not totally unexpected, as one can see for example in the recent work (13).

Our aim is now to present further results on the regular and chaotic dynamics of the Ziegler pendulum as defined in (10-11). In detail, we provide a proof of the non-density of the sets of periodic points in presence of the non-Hamiltonian symmetry that was conjectured in (11). Moreover, we analyze the coupling of two or more Ziegler pendulums in order to highlight the persistence of regular and chaotic dynamics in terms of mutual interactions. In particular, we look for a new control parameter for the non-Hamiltonian symmetry that generalize the qualitative and quantitative results presented in (11).

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