Dispersion and ellipticity of Rayleigh waves in a soil supporting resonant beams/plates

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Abstract

Elastic surface waves can propagate in a soil substrate hosting a periodic array of resonant elements. While early studies primarily explored this phenomenon in the GHz regime with resonators on the micrometer scale, extending these concepts to meter-scale structures shifts the focus to seismic frequencies. In this context, a network of beams on a soil substrate serves as a canonical configuration in seismology to study "site-city interaction," first introduced in pioneering works.Theoretical models of resonators coupled to soil typically employ simplified resonant systems with one or more degrees of freedom, leading to effective Robin-type boundary conditions for the soil itself. These models have been instrumental in advancing seismic metasurfaces, capable of efficiently shielding Rayleigh waves and Love waves. Early investigations of specific resonators, such as elastic beams or plates, initially focused on compressional resonances, followed by flexural resonances. Recently, the simultaneous consideration of both resonance types has been addressed, and the resulting models have been validated through scattering simulations.

Building on this foundation, we extend the analysis to surface waves in configurations involving three-dimensional beams and two-dimensional plates. Specifically, we demonstrate that the effect of resonant structures can be encapsulated in boundary conditions linking normal stress to displacements in the soil. These conditions depend on frequency and two dimensionless parameters: the slenderness of the elastic structures and a coupling parameter between these structures and the soil.The resulting dispersion relations reveal two types of surface waves. In the sagittal plane, hybridized Rayleigh waves emerge from the interplay of flexural and compressional resonances. Notably, within specific frequency ranges associated with flexural resonances, two Rayleigh waves can coexist, characterized by soil displacements exhibiting either high ellipticity or near-zero ellipticity. For three-dimensional beams, we also identify Love waves, associated with displacements perpendicular to the sagittal plane. These waves, sensitive only to flexural resonances, are the elastic analogs of plasmon polaritons in electromagnetism.

Reference :

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