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# Predicting the domain of linear elasticity of architected materials – focus on symmetries.

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## Abstract

Predicting the domain of linear elasticity of architected materials – focus on symmetries.

## Introduction

Modern engineering challenges have pushed forward the use of architected materials(9) and called for a new branch of material science to be explored. These special materials exhibit unconventional properties directly linked to their geometry. When composed of slender elements, architected materials can undergo large deformations exhibiting geometric nonlinearities through buckling or snapping behaviors of the cell walls(8). Furthermore, as any conventional material, their constitutive material could enter its non-linear domain(6). The combination if these two limits define the *domain of linear elasticity* of the architected material(7). This presentation will expose the domains of linear elasticity of various 2D architected materials and relate the symmetry of the domain to that of the parent material.

## Method

The domain of linear elasticity is constructed from the intersection of the non-buckling domain of the architected material and the linear elasticity domain of its constitutive material. It is well-known that buckling in architected materials can happen at different scales, especially when working with periodic architected materials(1)(4). Bloch-wave analysis is the appropriate tool for predicting buckling in such periodic materials(2)(5). This presentation will only consider 2D lattice periodic architected materials composed of beam elements that are modelled with Euler-Bernoulli beam theory. Moreover, the method only accounts for isotropic constitutive material and constant cross-section beams but the latter limitation can be lifted. The method could be extended to architected materials composed of plate or shell elements or to more complex modeling of the beam elements(3).

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## Results

The method has been validated on the classical examples of regular square, triangular and hexagonal honeycombs. The observed bifurcated patterns presented in the literature both numerically and experimentally are predicted by our method and results on the shape of the non-buckling domain can be retrieved.

More interestingly, it is possible to correlate the shape of the domain of linear elasticity with the symmetry of the parent architected materials. Based on several examples, it is observed that the symmetry group of the architected material is partially transferred to the symmetry group of the domain of linear elasticity, when observed in the deviatoric space. This can be explained by a tensorial representation of that domain.

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