Analytical and Numerical Analysis of Necking in Dynamic Expansion of Thin Structures

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Abstract

When exposed to high-velocity impacts or extreme dynamic stretching, thin structures made of aluminum, copper, and tantalum exhibit fragmentation behavior characterized by a range of fragment sizes and residual velocities (1). Since the 1940s (2), understanding the dynamic fragmentation of metallic structures under intense loading has been a major scientific challenge at the intersection of various military and civilian applications. Precisely predicting the fragment size distribution, fragment velocity, and the time to fracture remains an essential challenge in this domain.
To determine the characteristic dimensions of these fragments, we rely on Linear Stability Analysis (LSA) ((3-5)), which examines the response to small perturbations of a thin structure under extreme loading conditions. LSA enables the identification of the most unstable mode of perturbation. This analysis provides a foundational metric for predicting the scale of fragmentation patterns that emerge under dynamic loading.

In such a dynamic configuration, the combined effects of inertia and multiaxial stress within the neck region governs the instability development ((6-9)). Specifically, inertia stabilizes long-wavelength modes, while the multiaxiality of the flow stabilizes short-wavelength modes. Consequently, the most unstable mode has a wavelength that tends to fall within an intermediate range-a critical characteristic for accurately modeling fragmentation phenomena. Identifying and modeling this intermediate wavelength is essential for predicting fragmentation behavior, as it governs both the initiation and development of localized necking prior to complete material rupture.

Historically, Classical Linear Stability Analysis (CLSA) is based on the frozen coefficient theory and assumes an exponential growth of perturbations, facilitating the modeling of the early stages of instability under moderate dynamic loading conditions. However, this approach faces limitations in highly dynamic regimes. To overcome these limitations, an

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¶Corresponding author: christophe.czarnota@univ-lorraine.fr eXtended Linear Stability Analysis (XLSA) approach has been developed in (10-11) which relaxes the frozen coefficient approach and does not assume an exponential growth of instabilities. This model is expected to provide a more accurate and continuous description of the localization process. The XLSA allows us to follow the development of instabilities continuously over time, enabling the characterization of the most unstable mode and tracking the development of localized necking in thin metallic structures under dynamic extension.

In our research, the XLSA model is adopted to reproduce the local thinning development in cylindrical and annular metal structures (12). We supplement the XLSA approach with finite element simulations, providing a comparative basis for refining analytical predictions and validating model accuracy. Results indicate that both 1D and 2D-XLSA models produce reliable predictions for long-wavelength perturbations, while the 2D-XLSA model offers a more accurate evolution of short-wavelength instabilities. Leveraging insights from the 2D model, we propose to enhance the 1D model by modifying the multiaxial stress effects, thereby achieving predictions that align more closely with numerical observations. More precisely, a parametric study on the magnitude of the stress multiaxiality contribution is conducted showing that the Bridgman approach is not accurate for modes with intermediate to small wavelengths. While the most unstable wavelength remains largely not modified, the rate of development of the perturbation is highly sensitive to this contribution.

The combination of analytical modeling with finite element simulations, enables to improve the 1D approach, providing a more accurate description of the evolution of multiple necking during the early stages of the deformation.

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References:

(1) M. Altynova, X. Hu, G. S. Daehn, Increased ductility in high velocity electromagnetic ring expansion, Met. and Mat. Trans. 27A (1996) 1837–1844.

(2) N. F. Mott (1947), Fragmentation of shell cases, in : Proceedings of The Royal Society, Series A, pp. 300–308.

(3) C. Fressengeas and A. Molinari (1985). Inertia and thermal effects on the localization of plastic flow. *Acta Metallurgica*, 33(3):387–396.

(4) C. Fressengeas and A. Molinari (1994). Fragmentation of rapidly stretching sheets. *European Journal of Mechanics Series A Solids*, 13:251–251.

(5) V. B. Shenoy and L. B. Freund (1999). Necking bifurcations during high strain rate extension. *J. Mech. Phys. Solids*, 47:2209–2233.

(6) S. Mercier and A. Molinari (2003). Predictions of bifurcation and instabilities during dynamic extension. *Int. J. Solids Struct.*, 40:1995–2016.

(7) S. Mercier and A. Molinari, (2004). Analysis of multiple necking in rings under rapid radial expansion. *Int. J. Impact Eng.*, 30:403–419.

(8) S. Mercier, N. Granier, A. Molinari, F. Llorca et F. Buy, (2010). Multiple necking during the dynamic expansion of hemispherical metallic shells, from experiments to modelling. *J. Mech. Phys. Sol.*, 58(7):955–982.

(9) D. Jouve (2010). *Etude analytique de l'instabilité plastique de striction pour une plaque*

sollicitée en traction biaxiale. Thèse de doctorat, Ecole Polytechnique et Commissariat à l'Energie Atomique, France.

(10) M. Xavier, C. Czarnota, D. Jouve, S. Mercier, J. Dequiedt, A. Molinari, Extension of linear stability analysis for the dynamic stretching of plates : Spatio-temporal evolution of the perturbation, *European Journal of Mechanics-A/Solids* 79 (2020) 103860.

(11) M. Xavier, S. Mercier, S. Czarnota, S. El Maï, D. Jouve, J. L. Dequiedt, a. Molinari, Extension of 1d linear stability analysis based on the bridgman assumption. applications to the dynamic 19 stretching of a plate and expansion of a ring, *Int . J. Solids Struct.* 217-218 (2021) 215–227.
(12) S. El Maï, S. Mercier, J. Petit, A. Molinari, (2014). An extension of the linear stability analysis for the prediction of multiple necking during dynamic extension of round bar. *Int . J. Solids Struct.* 51, 3491-3507.