Hyperbolic formulation of gradient damage models and finite volume simulations of dynamic brittle fracture

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Abstract

In the context of "phase-field" approaches for brittle fracture based on a scalar damage internal variable, the well-posedness of mathematical formulations is in general ensured by means of geometrical or constitutive regularizations. The techniques of the former family such as the Thick Level Set or the Lip-field consist in constrain the damage variable geometrically while those of the latter are based on the introduction of the damage gradient in the internal energy. In both cases, however, the problem associated with damage evolution is governed by an elliptic partial differential equation whose numerical solution requires the solution of a global optimization problem. For dynamics applications (impact fragmentation, high-speed forming processes, etc.), the derived formulation prevents from using Godunov-type finite volume methods, which apply to hyperbolic PDEs and allow embedding some amount of the analytical solution within the numerical solution scheme in order to accurately track waves. Moreover, the global optimization problem of damage evolution may lead to prohibitive computational cost in any explicit scheme since it must be solved at each time step.

In this work, both previous points are addressed and a fully local hyperbolic formulation of gradient damage, well-suited to efficient explicit finite volumes computations, is proposed. The derivation of the governing equations under small strains is based on Hamilton's variational principle in which the solid internal energy is taken as a function of strains, the damage variable and the gradient of an additional non-dissipative damage variable. The latter, which can be seen as a micromorphic variable, is on the other hand involved in a 'microinertia term' associated with microvoids - following Frémond's model proposed in the 90s - that contributes to the kinetic energy of the solid. Therefore, non-local effects are carried by an auxiliary damage variable in a two-field formulation which makes a first significant difference with the already existing approaches to model dynamic brittle failure.

Hamilton's variational principle yields a set of conservative partial differential equations that does not account for irreversible effects. This system of first-order PDEs is hyperbolic

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providing that the second-order acoustic tensor has positive eigenvalues - the so-called strong ellipticity condition -, which is not automatically satisfied in presence of unilateral damage with total recovery of the elastic modulii due to microcracks closure.
However, the use of a deviatoric/hydrostatic additive decomposition of the fourth-order elasticity tensor in which a distinction between tension and compression stiffnesses is only made on the hydrostatic part allows ensuring hyperbolicity of the model. Finally, assuming the existence of a convex dual dissipation pseudo-potential, a rate-dependent damage evolution law is then derived as a Perzyna-like model and introduced as a source term in the governing system. The "default" formulation is non-dissipative, though irreversible, but the ratio between dissipation and storage of energy can be controled depending on the forms chosen for the internal energy and the yield function.

The explicit numerical solution of the derived hyperbolic system with source term can be made using a fractional-step method composed of the following steps in a staggered fashion:
1/ Solve the homogeneous part by means of a Godunov-type finite volume scheme. This part can actually be split into two uncoupled problems related to a purely elastic part on the one hand, and the transport of the microdamage on the other hand.
2/ Solve an ordinary differential equation system associated with the source term and for which initial conditions result of the homogenous problem. It is worth noticing that in order to avoid any influence on the CFL condition, a backward Euler discretization coupled with a Newton–Raphson method may be preferred.
Such a procedure can be seen as a predictor–corrector approach in which damage flow can only occur in the ODE.

Several numerical results show very promising features of the formulation.
First, good convergence properties are exhibited on a one-dimensional multifragmentation test. Namely, the number of fragments with respect to the strain rate imposed to a one-dimensional domain for a given mesh as well as the number of fragments with respect to the number of elements for a given imposed strain rate. In addition, the complexity of the numerical scheme used is of the order of the square of the elements length, which cannot be achieved with the other existing nonlocal formulation of damage.
Second, consistent crack paths are observed for the two-dimensional Kalthoff–Winkler test.
At last, the model allows capturing crack branching in perfectly symmetrical two-dimensional settings.