The deterministic role of edge states in slender structures buckling, and how edge states depend on geometric defects

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Abstract

The buckling of slender elastic structures, particularly the buckling of spherical and cylindrical shells, is highly sensitive to imperfections and external perturbations. This often leads to buckling at loading conditions well below the threshold for linear instability. One approach to rationalize this phenomenon is to consider the nonlinear finite-amplitude instability of the linearly stable system. Within such a dynamical systems description and for subcritical values of the control parameter (typically the applied load or end-shortening), the unbuckled state is surrounded by its basin of attraction. Initial perturbations within the basin decay to the unbuckled state, while perturbations outside trigger buckling. The basin boundary is formed, at least locally, by the stable manifold of special unstable equilibria, termed *edge states*. Consequently, these edge states control the critical size of deformation perturbations triggering buckling.

Edge states and their dependence on loading conditions have been numerically analyzed for axially compressed cylinder shells in the absence of geometric defects, where their role in guiding buckling has been demonstrated (1, 2). For imperfect cylinder shells with geometric imperfections, edge state equilibria have so far not been studied numerically in detail. However, experiments on the buckling of commercial aluminum shells strongly suggest that (a) edge states remain relevant for the buckling of imperfect shell structures and (b) edge states may encode the affects of specific realizations of geometric defects so that tracking edge states can provide information on the spontaneous buckling load of an individual defected shell (3). Here, we consequently study the role of edge states in the buckling of both geometrically perfect and imperfect slender structures.

For a perfect axially loaded cylinder shell, we demonstrate the existence of a full snakes-and-ladders bifurcation structure. Two branches of unstable equilibria containing either an even or odd number of localized dimples undergo sequences of saddle-node bifurcations under parametric continuation in the imposed end-shortening. These branches of reflection-symmetric solutions are connected by asymmetric 'rung' branches. Those rungs bifurcate off the snaking branches in pitchfork bifurcations that break the discrete reflection symmetry. Together snaking and rung branches form the well-known snakes-and-ladders bifurcation structure, previously studied within the Swift-Hohenberg model PDE with 2-3 nonlinearity.

Geometric imperfections, small modifications of the unloaded reference configuration, may break continuous and discrete symmetries and are thus expected to modify the edge state

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equilibria as well as their bifurcation structure. We thus develop numerical tools for the parametric continuation of equilibria in the amplitude of geometric imperfections. These methods are developed in the context of a circular arc subject to a central point load. We show that even for a large class of imperfections, there is a unique edge state that captures critical deformations to trigger buckling and that this edge state can be identified using probing techniques such as those developed in the context of shell buckling studies. Transferring the parametric continuation scheme, termed *Continuation in Geometry (CIG)*, to the axially loaded cylinder shell problem, we study the imperfection-dependence of edge state equilibria. We specifically discuss the breakdown of regular snaking, the emergence of additional solution branches with complex spatial structure, and the modification of the critical load value, at which edge-state branches bifurcate off the trivial unbuckled equilibrium. The significance of characterizing the imperfection dependence of edge states lies in the potential to better understand and predict when a real defected slender structure buckles and collapses.

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