
Consistent Eulerian and Lagrangian variational formulations of non-linear kinematic hardening

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Abstract

Numerical simulation of impacts on solids generally consider a constitutive response that embeds, amongst others, an elastic-plastic contribution with a sole isotropic hardening. Indeed, the latter is sufficient in case of monotonic loadings. However, various engineering applications may also involve repeated impacts, like Laser Shock Peening used for instance for mitigating back stresses in rolled stainless sheets (1), or cyclic impacts sequentially applied in forward and reverse directions. In these cases, eventual local cyclic loadings can yield Bauschinger and/or ratchetting effects, such that considering non-linear kinematic hardening in the constitutive response becomes important.

Two Eulerian and Lagrangian hyperbolic modelings of thermo-hyperelastic-plastic solids coupled with a non-linear kinematic hardening in finite strains are proposed in this work (2). Constitutive equations are written in a consistent manner between both kinematical descriptions, first via the definition of back stresses, second thanks to plastic evolutions laws written in the respective settings, rather than in the intermediate one. The constitutive equations in the two settings are coupled with their respective multi-field first order systems of balance equations, so that the well-posedness of the Eulerian and Lagrangian hyperbolic modelings are ensured.

The structure of the constitutive modelings then naturally allows to build variational formulations of the dissipative thermo-mechanical local constitutive problem written in both Eulerian and Lagrangian descriptions. First order accurate variational constitutive updates are then derived from their continuous counterparts. Following the framework developed in (3), they admit updated values of specific internal energy as input data in place of the temperature. This makes these integrators naturally compatible with the solution of the conservation of total energy. However, constitutive equations written in initial and current settings now require a novel writing of discrete Euler-Lagrange equations, and a new parameterization of the flow direction.

Space discretization of balance equations is here performed via the second order accurate flux difference splitting finite volume method, embedding non-conservative fluxes arising in the Eulerian formulation. A sequence of test cases performed at one material point, then on 1D and 2D examples allow to show on the one hand the good agreement between solutions computed with the two descriptions, and on the other hand the truly different predicted

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solutions when accounting for kinematic or isotropic hardenings in case of a cyclic loading.

References

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