A variational approach to boundary effects in higher-order homogenization

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Abstract

From an engineering point of view, it is convenient to describe composite materials using homogeneous effective properties. When the microstructure is periodic, asymptotic homogenization is particularly well suited for this aim. Classical homogenization corresponds to the dominant order model and yields an effective standard Cauchy medium. At next orders, we can derive additional corrections that depend on the successive strain gradients. These corrections are typically of interest to capture size-effects appearing for microstructures with contrasted stiffness properties.

However, these higher-order models present two major limitations. First, the corrections produced by homogenization can handle size-effects that occur in the bulk region, but are not suited to the analysis of the boundaries. In fact, they miss significant boundary effects which can ruin the quality of the predictions. Secondly, these higher-order models present several mathematical inconsistencies, including non-positive strain-gradient stiffnesses. As a result, the effective energy is not necessarily positive and any equilibrium solution is unstable with respect to short-scale oscillations. To handle these two limitations simultaneously, we elaborate a new homogenization procedure that includes boundary effects. By contrast with usual approaches, in our procedure the homogenization is carried at the energy level, rather than on the strong form of the equilibrium. Besides, the positivity of the resulting energy is guaranteed by an original truncation method (1).

As an example, we consider a 2D laminate that is invariant in the direction perpendicular to the fibers. The resulting effective energy contains a bulk term that is positive, plus a boundary term that accounts for the energy generated by the boundary effects. The effective stiffness on boundary is computed numerically, by combining solutions of elementary 2D elastic problems formulated on a semi-infinite strip. We show that, by contrast with usual asymptotic homogenization, this higher-order model is able to capture size-effects occurring in the interior domain, as well as near the boundaries.

(1) M. Thbaut, B. Audoly, and C. Lestringant. Fixing non-positive energies in higher-order homogenization. 2024.

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