
Thermodynamic potentials for viscoelastic composites with simple constituent rheologies

Martín Idiart*¹

¹Universidad Nacional de La Plata/CONICET – Argentina

Abstract

The viscoelastic response of composite media is the result of intricate interactions between elastic and viscous deformation processes operating within the different constitutive phases. A key consequence of such interactions is that microscopic constitutive descriptions based on finite sets of internal variables give rise to macroscopic constitutive descriptions with an infinity of internal variables. This seemingly ineludible feature of multiscale inelasticity has hindered the development of a mean-field thermodynamic theory of viscoelastic composites that correlates macroscopic and microscopic energetics in a rational and practical manner on a par with existing mean-field thermodynamic theories of purely elastic and purely viscous composites. Knowledge of such correlations is required, for instance, to adequately quantify the contribution of microscopic viscous dissipation to heating during any thermomechanical process. Mean-field thermodynamic potentials are derived in this work by resorting to variational approximations. Efforts to do so are not new. Central to the present effort, however, is the recognition that any viscous deformation field—which does not necessarily comply with any differential constraints— can be additively decomposed into an irrotational field and a solenoidal field—which do comply with differential constraints—in such a way that variational approximations available for elastic potentials become applicative to viscoelastic potentials. The variational approximations already available for random elastic media prove then instrumental. Attention is restricted to composites with ellipsoidal microstructural randomness and with constituents exhibiting Maxwellian or Kelvinian rheologies. The resulting thermodynamic potentials simultaneously i) conform to the two-potential formalism with a finite number of effective internal variables with explicit physical meaning, ii) comply with the convexity required by the generalized standard model, iii) can be expected to comply in a certain sense with elementary bounds, iv) predict the expected purely elastic and purely viscous regimes, v) can improve on decoupled descriptions of viscoelastic transitions, vi) can be realizable under certain circumstances, and vii) are amenable to including nonlinearly viscous effects. Specific approximations of the Hashin-Shtrikman and the Self-Consistent types are worked out in detail. Intraphase statistics of the stress field up to second order are also provided. Finally, extensions to nonlinear material behavior are discussed. It is anticipated that results and procedures reported in this work will bear relevance to open questions in the viscoelasticity theory of composite media.

*Speaker