
On the use of the Hamiltonian invariant of elastic rods

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Abstract

The static-dynamic analogy discovered by G. Kirchhoff shows that the statics of an elastic beam are equivalent to the dynamics of a spinning top. In this analogy, time and angular velocity are, for example, equivalent to arc length and curvatures. This static-dynamic analogy allows us to find a quantity that is invariant along the elastic rod at equilibrium: A spinning top will have its mechanical energy constant in time and, in the same manner, an elastic rod will have the sum of its curvature energy and its axial force uniform along the structure. The invariant property is known in simple cases, but the present approach generalises it to more complex cases where extensibility, shear, gravity, and contact are involved. Finally, we will show how to compute this invariant and illustrate its usefulness in three examples: the capstan, the elastic arm scale, and plectonemes.

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